

1) Derive abba

S	initial
ABS	①
ABABS	①
ABAB	①b
ABBA	②
abba	④, ⑤ *

Derive babbaaab

S	initial
ABS	①
ABABS	②
ABABABS	③
ABABABABS	④
ABABABAB	⑤
BAABABAB	⑥
BABAABAB	⑦
BAABAAB	⑧
BABBAAB	⑨
babbaaab	⑩ *

Prove EQUAL

- ① Introduces exactly one A and one B
- ② Doesn't change # of A's or B's
- ③ Doesn't change # of A's or B's
- ④ and ⑤ convert from uppercase to lowercase
- ⑥ and ⑦ merely rearrange order of a's & b's

2) ① Relabel S from EQUAL as E

② Introduce new S-state:

$$S \rightarrow AS \mid AE$$

DONE

3) ① Relabel S from MOREA as M

② Introduce new N-state:

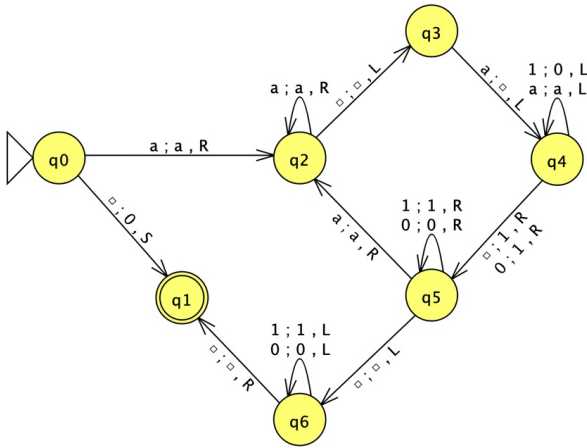
$$N \rightarrow BN \mid BE$$

③ Introduce new S-state:

$$S \rightarrow M \mid N$$

DONE

4) accepts number in unary and converts to binary



- q<sub>2</sub> - go to end
- q<sub>3</sub> - erase rightmost a
- q<sub>4</sub> - rewind to Δ or 0  
carry if necessary
- q<sub>5</sub> - fast forward to next a
- q<sub>6</sub> - no more a's - rewind to beginning

5) Describe unary exponentiation

a # b-part b e-part #

- 1) start by writing a # before the base and exponent  
write a # after the e-part
- 2) if e-part empty, erase back until # seen; rewind to beginning; done!
- 3) decrement e-part by one
- 4) multiply lhs-of-# with b-part and write result to the right of the #
- 5) move/overwrite rhs-of-# to lhs-of-##
- 6) goto step #2

6)

$$m \Rightarrow n \Rightarrow f \Rightarrow x \Rightarrow m(n(f))(x)$$

$$n \Rightarrow f \Rightarrow x \Rightarrow N2(n(f))(x)$$

$$f \Rightarrow x \Rightarrow N2(N3(f))(x)$$

$$x \Rightarrow N2(N3(\text{succ}))(x)$$

$$N2(N3(\text{succ}))(0)$$

$$N3(\text{succ})(N3(\text{succ})(0))$$

$$N3(\text{succ})(\text{succ}(\text{succ}(\text{succ}(0))))$$

$$N3(\text{succ})(3)$$

$$\text{succ}(\text{succ}(\text{succ}(3)))$$

6

$$m \rightarrow N2$$

$$n \rightarrow N3$$

$$f \rightarrow \text{succ}$$

$$x \rightarrow 0$$

$$N2(\lambda)(z) \rightarrow \lambda(\lambda(z))$$

$$N3(\text{succ})(0) \rightarrow \text{succ}(\text{succ}(\text{succ}(0)))$$

$$\text{succ}(\text{succ}(\text{succ}(0))) \rightarrow 3$$

$$N3(\text{succ})(3) \rightarrow \text{succ}(\text{succ}(\text{succ}(3)))$$

$$\text{succ}(\text{succ}(\text{succ}(3))) \rightarrow 6$$