CSC1 Homework #10

1) Do the following Grammars Generate Any Words?

i

$S \to aSa$
$S \to bSb$

Convert to CNF

$S \to AX$ $\quad S \to aX$
$S \to BY$ $\quad S \to bY$
$X \to SA$ $\quad X \to Sa$
$Y \to SB$ $\quad Y \to Sb$
$A \to a$
$B \to b$

↓ No string of terminals...

No words Produced

ii

$S \to XY$
$S \to SY$
$X \to SY$ ⊗
$X \to a$ ①
$Y \to SX$
$Y \to b$

In CNF

$S \to aY$
$S \to SY$
$Y \to Sa$ ⊗
$Y \to b$ ②

$S \to ab$
$S \to SY$

S eliminated "ab" produced

iii

$S \to AB$
$A \to BC$ ⊗
$A \to b$ ①
$B \to CD$
$C \to DA$
$D \to a$

In CNF

$S \to aB$
$B \to CD$
$C \to Db$
$D \to a$ ②

$S \to aB$
$B \to Ca$
$C \to ab$ ③

$S \to aB$
$B \to aba$ ③

$S \to aaba$

↓

S eliminated "aaba" produced

iv

$S \to XS$
$X \to YX$ ⊗ ①
$X \to a$
$Y \to YY$
$Y \to XX$

In CNF

$S \to aS$
$Y \to YY$ ⊗
$Y \to aa$ ②

$S \to aS$

↓

No string of terminals...

No Words Produced

v

$S \to AB$
~~$A \to BSB$~~ $\quad A \to BR_1$ ⊗
$A \to CC$ ⊗ $\quad R_1 \to SB$
$A \to a$ ⊗
$A \to b$ ①
~~$B \to AAS$~~ $\quad B \to AR_2$
$B \to CC$ $\quad R_2 \to AS$
$C \to SS$
$C \to b$
$C \to bb$

Convert to CNF

$S \to bB$
$R_1 \to SB$
$B \to bR_2$
$R_2 \to bS$
$B \to CC$
$C \to SS$ ⊗
$C \to b$ ②
$C \to bb$ ⊗

$S \to bB$
$R_1 \to SB$
$B \to bR_2$ ⊗
$R_2 \to bS$
$B \to bb$ ③

↓

$S \to bbb$ ④
$R_1 \to Sbb$
$R_2 \to bS$

Eliminated S "bbb" produced

2. Decide whether the Following Grammars Accept Finite or Infinite Languages

i

- ✗ $S \to XS$
- $S \to b$
- $X \to YZ$ ← useless
- $Y \to ab$
- ✗ $Z \to XY$

S only yields "b"
∴ finite Language

ii

- ✗ $S \to XY$
- $S \to bb$
- ✗ $X \to YX$ ← useless
- ✗ $Y \to XY$
- $Y \to SS$

$S \to bb$
$Y \to SS$

S is not self-embedded
∴ finite language

iii

- $S \to XY$
- ✗ $X \to AA$
- $X \to YY$
- $X \to b$
- ✗ $A \to BC$ } useless
- ✗ $B \to AC$ } useless
- ✗ $C \to BA$ } useless
- $Y \to a$

$S \to XY$ ← doesn't S.E.
$X \to YY$ ← doesn't S.E.
$X \to b$
$Y \to a$

Finite Language

iv

- $S \to XY$
- ✗ $X \to AA$
- $X \to XY$
- $X \to b$
- ✗ $A \to BC$ } useless
- ✗ $B \to AC$ } useless
- ✗ $C \to BA$ } useless
- $Y \to a$

$S \to XY$ ← doesn't S.E.
$X \to XY$ ← does S.E.
$X \to b$
$Y \to a$

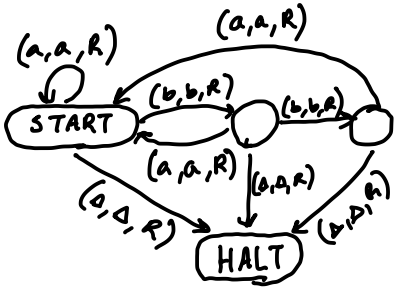
Infinite Language

v

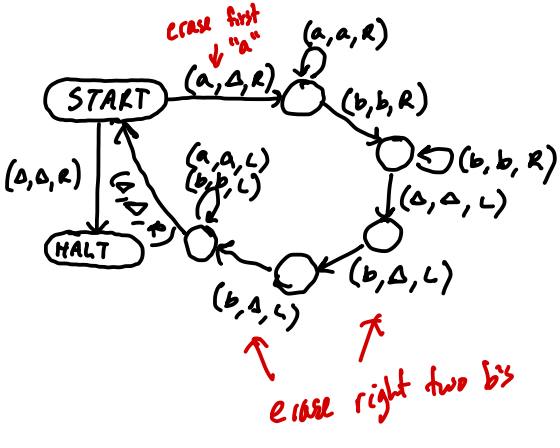
- $S \to SS$
- $S \to b$
- $X \to SS$ } useless
- $X \to SX$ } useless
- $X \to a$

S self-embeds
∴ Infinite Language

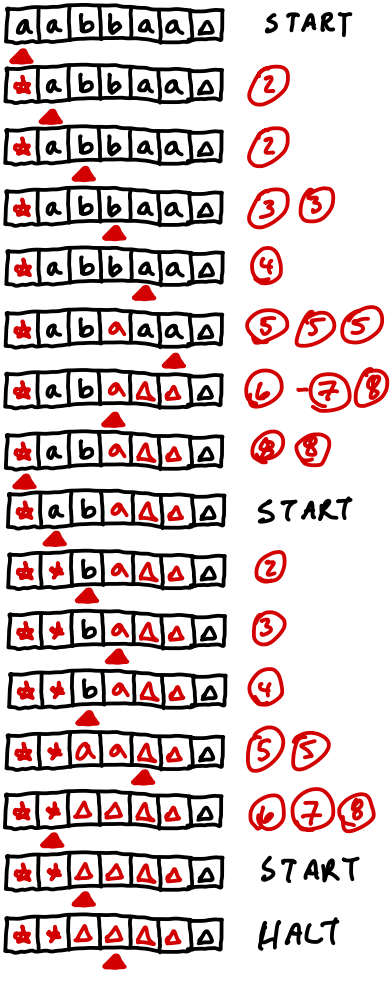
3. TM accepting all words not containing "bbb"



4. Build a TM that accepts $a^n b^{2n}$



5) Trace aabbaa on Slide 11



6) Trace aabbaa on slide 7