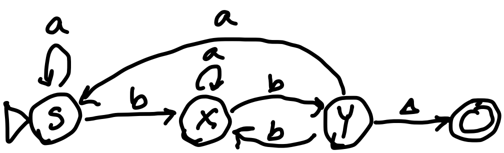
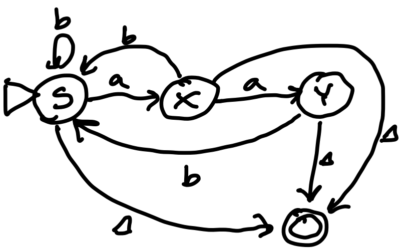
CSCI 340 Homework 8

1) A. All strings that end in b and have an even number of b's



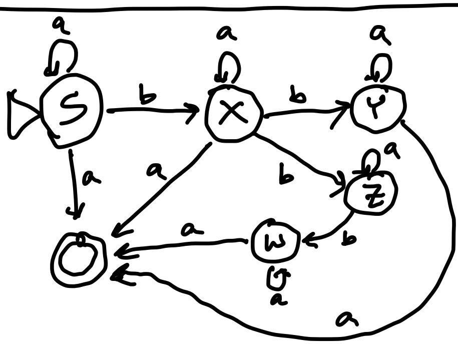
$S \to aS \mid bX$
$X \to aX \mid bY$
$Y \to aS \mid bX \mid \Lambda$

B. All strings without the substring aaa



$S \to bS \mid aX \mid \Lambda$
$X \to bS \mid aY \mid \Lambda$
$Y \to bS \mid \Lambda$

2)



All strings that contain less than 4 b's and end in an a.

$(a^* + ba^* + ba^*ba^* + ba^*ba^*ba^*)a$

no b's | 1 b | 2 b's | 3 b's

3)
$S \to (S)$
$S \to S+S$
$S \to S^*$
$S \to SS$
$S \to a$
$S \to b$

This language is <u>not</u> regular. Consider only the production $S \to (S)$. This cannot be converted to a semiword

4) $S \to XY$
$X \to aX \mid Xa \mid a$
$Y \to bY \mid b$

X has a production of the form $N \to Nt$. Regular grammars only have semiwords. Since X represents one-or-more a's, we can remove the "malformed" production

5) $S \to XaX$
$S \to bX$
$X \to XaX$
$X \to XbX$
$X \to \Lambda$

$S \to Xa$
$S \to aX$
$S \to b$
$X \to Xa$
$X \to aX$
$X \to Xb$
$X \to bX$

6) $S \to aX \mid Yb$
$X \to S$
$Y \to bY \mid b$

$X \to aX$
$X \to Yb$

7) $E \to E+E \;\to\; E \to EPE$
$E \to E*E \;\to\; E \to EME$
$E \to (E) \;\to\; E \to LER$

Replace terminals with non-terminals

2

$E \to 7$

1

$P \to +$
$M \to *$
$L \to ($
$R \to)$

terminal productions

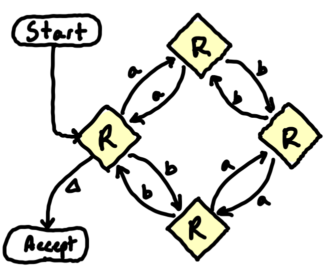
$E \to ER_1$
$R_1 \to PE$
$E \to ER_2$
$R_2 \to ME$
$E \to LR_3$
$R_3 \to ER$

3 Split 3+ NT with R-states

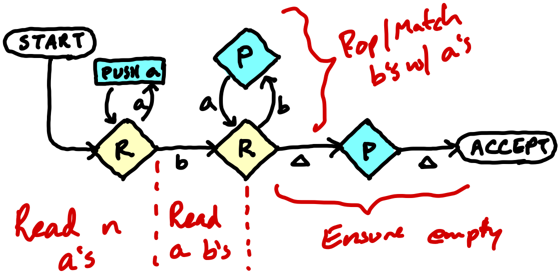
Final Grammar

$E \to ER_1 \mid ER_2 \mid LR_3 \mid 7$
$R_1 \to PE$
$R_2 \to ME$
$R_3 \to ER$
$P \to +$ $L \to ($
$M \to *$ $R \to)$

8) We can just convert the FA to a PDA



9)



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10)

Tape: ~~a~~ ~~a~~ ~~a~~ ~~b~~ ~~b~~ ~~b~~ ~~Δ~~

Stack: ~~a~~ ~~a~~ ~~a~~

1) START
2) $READ_1$ a
3) PUSH a
4) $READ_2$ a
5) PUSH a
6) $READ_2$ a
7) PUSH a
8) $READ_2$ b
9) POP_1 a
10) $READ_3$ b
11) POP_1 a
12) $READ_3$ b
13) POP_1 a
14) $READ_3$ Δ
15) POP_2 Δ
16) ACCEPT

Tape: ~~a~~ ~~a~~ ~~a~~ ~~a~~ ~~b~~ ~~b~~ Δ

Stack: a a ~~a~~ ~~a~~

1) START
2) $READ_1$ a
3) PUSH a
4) $READ_2$ a
5) PUSH a
6) $READ_2$ a
7) PUSH a
8) $READ_2$ a
9) PUSH a
10) $READ_2$ b
11) POP_1 a
12) $READ_3$ b
13) POP_1 a
14) $READ_3$ Δ
15) POP_2 a
16) REJECT

CFG

$S \to aSa$

$S \to aSb$

$S \to ab$

English

First "half" is all a's. Second "half" must start with a b

CSCI 340 Homework #9

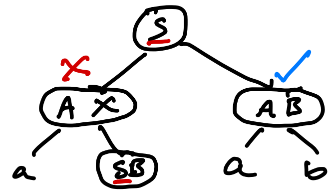
1) $L = \{a^n b^n\}$

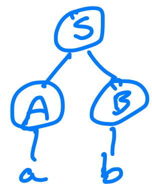
$S \to aSb$
$S \to \Lambda$

$\Longrightarrow$

Chomsky'd

$S \to AX$
$S \to AB$
$X \to SB$
$A \to a$
$B \to b$





Is the only derivation tree with no self embedding states

2) If Palindrome is odd length, let x be the middle letter and v & y be the adjacent characters.

If Palindrome is even length, let x be the middle two letters and v & y be the adjacent characters.

3) The regular language pumping lemma tries to show $w = xyz \therefore xy^n z \in \mathcal{L}$

The context-free pumping lemma tries to show $w = uvxyz \therefore uv^n x y^n z \in \mathcal{L}$

Since $\Sigma = \{a\}$, RL y^n can <u>always</u> equal CF $v^n y^n$
RL x can <u>always</u> equal CF ux
RL z can <u>always</u> equal CF z

4) Recursive Dfn for Regular Expressions:

1) a is in RE
2) b is in RE
3) if R is in RE_1 so is R^*
4) if R is in RE_1 so is (R)
5) if R_1 and R_2 are in RE, so is $R_1 + R_2$
6) if R_1 and R_2 are in RE, so is $R_1 R_2$

1 & 2 are context free because a terminal by itself is context free

$S \to a$
$S \to b$

3 is context free by the theorem of closures

4 is context free by the following production:

a) Relabel S as S'
b) $S \to (S')$

5 is context free by the theorem of union

6 is context free by the theorem of concatenation

$\square$

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5) A

$S \to S_1 \mid S_2$
$S_1 \to a S_1 b$
$S_1 \to \Lambda$
$S_2 \to a X_2$
$X_2 \to a X_2 \mid b X_2 \mid \Lambda$

B EVEN-EVEN* is still EVEN-EVEN

$S \to aaS \mid bbS \mid abX \mid baX \mid \Lambda$
$X \to aaX \mid bbX \mid abS \mid baS$

C $S \to S_1 S_2$
$S_1 \to a S_1 b \mid a \mid b$
$S_2 \to a S_2 b \mid \Lambda$

6) Split b^y into $b^x b^z$

$$\underbrace{a^x b^x}_{a^n b^n} \mid \underbrace{b^z a^z}_{b^n a^n}$$

$S \to aSb \mid \Lambda$ $\quad S \to bSa \mid \Lambda$

$S_1 \to aS_1b \mid \Lambda$ $\quad S_2 \to bS_2a \mid \Lambda$

$$\boxed{\begin{array}{l} S \to S_1 S_2 \\ S_1 \to aS_1b \mid \Lambda \\ S_2 \to bS_2a \mid \Lambda \end{array}}$$

7)

A count of a's in $a^n b^n a^n$ is always different than the count of b's.

The resulting language must be empty

B PALINDROME can be odd or even length. If it's even length, then it must also be even-even. Therefore, the resulting language <u>must</u> be odd palindrome

C No palindrome is of the form $a^n b^n$. Because of this fact, $\{a^n b^n\}$ is a superset of PALINDROME. Therefore, the language is still palindrome