CSCI 340: Computational Models

CFG = PDA

Chapter 15

Department of Computer Science

Building a PDA for Every CFG

Theorem

Given a CFG that generates the language L, there is a PDA that accepts exactly L

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Given a PDA that accepts the language L, there exists a CFG that accepts exactly L

Both of these theorems were discovered independently by Schützenberger, Chomsky, and Evey

CFG to PDA Algorithm

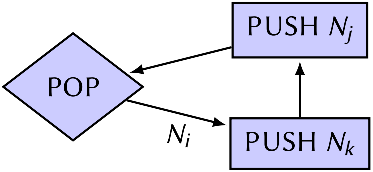
Note: We assume the CFG grammar is defined in CNF

$$X_1 \rightarrow X_2X_3$$
$$X_1 \rightarrow X_3X_4$$
$$X_2 \rightarrow X_2X_2$$
$$\ldots$$
$$X_3 \rightarrow a$$
$$X_4 \rightarrow a$$
$$X_5 \rightarrow b$$
$$\ldots$$

Two forms:

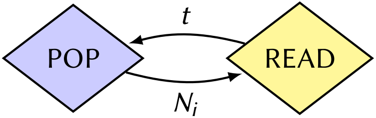
$$N_i \rightarrow N_jN_k$$
$$N_i \rightarrow t$$

Handling form $N_i \rightarrow N_jN_k$:



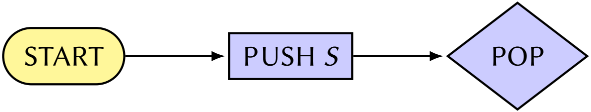
Note: non-terminals are pushed in reverse order

Handling form $N_i \rightarrow t$:

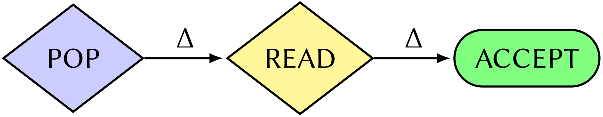


CFG to PDA Algorithm

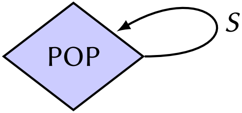
Start of machine:



End of machine:



If a language should accept λ, include:

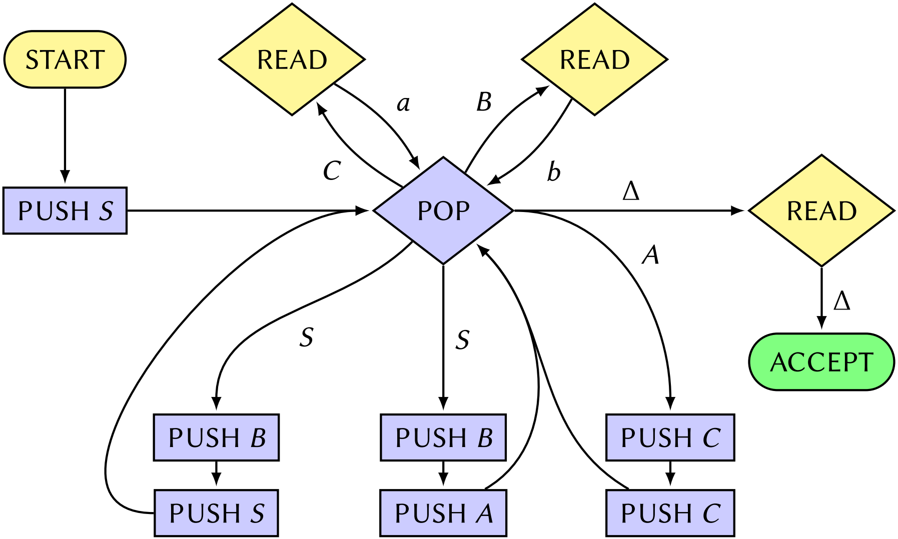


Example

Consider the following grammar (in CNF):

$$S \longrightarrow SB$$
$$S \longrightarrow AB$$
$$A \longrightarrow CC$$
$$B \longrightarrow b$$
$$C \longrightarrow a$$

Example


START
PUSH S
READ
a
C
READ
B
b
POP
Δ
READ
A
Δ
ACCEPT
S
S
PUSH B
PUSH S
PUSH B
PUSH A
PUSH C
PUSH C

PDA to CFG

"This is a long proof by constructive algorithm. In fact, it is unquestionably the most torturous proof in the book; parental consent is required"

Pages 327 – 347

PDA to CFG

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