1. Consider the language $S^*$, where $S = \{aa \ b\}$. How many words does this language have of length 4? of length 5? of length 6? What can be said in general?

2. Consider the language $S^*$, where $S = \{aa \ aba \ baa\}$. Show that the words $aabaa$, $baaabaa$, and $baaaaaababaaaa$ are all in this language. Can any word in this language be interpreted as a string of elements from $S$ in two different ways? Can any word in this language have an odd total number of a’s?

3. Prove that for all sets $S$,
   (a) $(S^+)^* = (S^*)^*$
   (b) $(S^+)^* = S^*$
   (c) Is $(S^*)^* = (S^+)^*$ for all sets $S$?

4. Using any recursive definition of the set EVEN, show that all the numbers in it end in the digits 0, 2, 4, 6, or 8

5. Show that if $n$ is less than 31, then $x^n$ can be shown to be in POLYNOMIAL in fewer than eight steps

6. Give a recursive definition for the language PALINDROME. Make sure it works for even and odd length strings. $\Sigma = \{a \ b\}$

7. Give recursive definitions for the following languages. $\Sigma = \{a \ b\}$
   (a) the language EVENSTRING of all words of even length
   (b) the language ODDSTRING of all words of odd length
   (c) the language AA of all words containing substring aa
   (d) the language NOTAA of all words not containing substring aa