MAKING QUANTUM COMPUTING ACCESSIBLE: A PATH FOR CS MAJORS WITH LIMITED FOUNDATIONS

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ABSTRACT

As quantum computing continues to advance, its transformative potential in fields such as cryptography, material science, and machine learning is undeniable. However, incorporating quantum computing into computer science curricula remains a significant challenge due to the steep learning curve posed by quantum mechanics and the heavy reliance on physics concepts. To address this, we propose an approach to quantum computing education tailored for computer science students, aimed at making complex concepts accessible through a focus on linear algebra. In this paper, we summarize nine key linear algebra concepts essential for understanding quantum computing and express the four postulates of quantum mechanics from a linear algebra perspective. Additionally, we provide practical materials and hands-on resources that educators can easily adapt for their own courses, fostering broader adoption of quantum education. By lowering barriers to entry, this work empowers both students and educators to engage with quantum computing, helping to prepare a capable workforce for this transformative technology.

1 Introduction

Quantum computing leverages the principles of quantum mechanics, such as superposition and entanglement, to process information in fundamentally different ways compared to classical computing (see [1] for a detailed introduction). In recent years, the field has advanced rapidly, with significant progress in both theoretical research and practical applications. In December 2024, Google's parent company, Alphabet, introduced the Willow quantum computing chip, which claims to solve a problem in five minutes, a task that would take classical computers an impractically long time to complete. Similarly, in November 2024, IBM unveiled its most advanced quantum computers, suggesting enhanced computational capabilities and potential breakthroughs in fields such as cryptography, material science, and machine learning. Beyond research laboratories, industries are investing heavily in quantum technologies, with applications in cryptography, material science, and machine learning driving this momentum. As quantum computing moves from theoretical research into practical applications, the need to educate the next generation of computer scientists has become a strategic priority (see [2] and [3]).

Despite its significance, quantum education remains in its infancy (see [3]). While some universities have introduced quantum courses, the subject has yet to be broadly integrated into standard computer science curricula. This lack of widespread adoption is partly due to the steep learning curve associated with quantum mechanics, a discipline traditionally grounded in physics. Concepts such as the spin of electrons, the behavior of particles at quantum scales, and wave-particle duality require a level of familiarity with physics that most computer science undergraduates lack. These challenges are compounded by the use of specialized notations and terminologies that can overwhelm students without prior exposure.

Yet, the urgency to overcome these barriers cannot be overstated. As quantum technologies continue to mature, the demand for a workforce capable of designing, programming, and applying quantum systems will only grow. The democratization of quantum education is essential to meet this need and ensure that a diverse array of students can participate in shaping this transformative field.

By taking a computer science perspective, this approach makes quantum computing concepts more accessible, significantly reducing the need for extensive physics knowledge. This paper not only highlights key foundational concepts, but also provides hands-on materials and resources that educators can easily adapt to create their own quantum computing courses. With this framework, we aim to lower barriers and empower educators to initiate quantum computing education within the computer science community. This effort represents a crucial step toward making quantum computing both practical and inclusive, ensuring that it reaches a broader audience of students and educators.

2 Related Work

Quantum computing education is an evolving field that is gaining considerable attention in academic settings, particularly in computer science programs. The recent literature emphasizes innovative teaching approaches and curricula designed to make quantum concepts more accessible to students.

Several studies have focused on the integration of quantum computing into existing computer science programs. For instance, the development of modular and scaffolded learning frameworks helps students progressively understand complex topics, reducing the intimidation often associated with quantum mechanics. These frameworks facilitate the incremental acquisition of knowledge, allowing students to seamlessly connect classical computer science principles with quantum theories [4], [5].

Moreover, hands-on experience with real quantum computing platforms has become an essential component of effective instruction. Recent education initiatives encourage students to use cloud-based quantum computing platforms like IBM Q and Microsoft Quantum Development Kit, where they can run experiments and manipulate quantum circuits. This experiential learning not only reinforces theoretical concepts, but also prepares students to tackle real-world challenges posed by quantum technologies [6].

Game-based learning and simulations have been shown to improve participation among students learning about quantum computing. Studies illustrate that integrating playful elements into the curriculum, such as serious games designed to teach quantum principles, can significantly improve the understanding and retention of students of complex concepts while fostering enthusiasm for the subject matter [7], [8].

Furthermore, research emphasizes the importance of interdisciplinary approaches in quantum computing education. By engaging students from various academic backgrounds, including physics, mathematics, and computer science, educators can create a richer learning environment that promotes collaborative techniques to understand quantum computing [9].

Lastly, there is a growing focus on equity and inclusivity within quantum computing education. Recent articles highlight initiatives aimed at attracting underrepresented groups into STEM fields, specifically quantum technology. Tailored outreach programs and mentorship opportunities are essential to build a diverse pipeline of future quantum computing professionals and to ensure that the field benefits from a wide range of perspectives [10].

The findings from these studies collectively illustrate that by employing diverse pedagogical strategies, promoting practical experiences, and emphasizing inclusivity, educators can effectively prepare students for the future of quantum technologies.

3 Pedagogical Approach and Essential Concepts

Traditional quantum computing courses often introduce fundamental concepts like superposition and entanglement through physical examples, such as photon polarization or electron spin. These approaches, rooted in physics, require students to grasp additional concepts, such as electromagnetic waves and quantum measurements, which can be challenging for computer science students without a solid background in physics (for example, see [11]).

In contrast, this paper takes a linear algebra-based approach, similar to [4], drawing on concepts familiar to most computer science majors. By presenting quantum computing as a generalization of classical probabilistic computing and abstracting physical phenomena through linear algebra, the course minimizes the need for specialized physics knowledge. As described in [4], concepts like superposition and entanglement are framed as properties of unit vectors in Hilbert spaces, and quantum gates are introduced as simple operations on these vectors, avoiding the need for complex number manipulation.

One key distinguishing feature of our approach is its focus on computational theory rather than quantum programming. Given the limited commercial success of quantum programming languages and their experimental nature, the course emphasizes the computational parallels between classical and quantum devices [12], [13] and [14]. This approach not only deepens understanding of quantum complexity classes but also enhances the connection to classical computation theory. By emphasizing these core theoretical concepts, the course helps learners appreciate the foundational principles of classical computation and recognize its computational limits. This perspective allows for a deeper comprehension of both classical and quantum computing, reinforcing their interrelationship. Focusing on theory, rather than experimental programming languages, provides a more practical and meaningful introduction to quantum computing.

In the following, we outline some of the most fundamental topics covered in our approach. These concepts are designed to be accessible, requiring only a minimal mathematical background, especially in the early stages. As learners progress, they gradually build upon this foundation, enhancing both their understanding of quantum computing and their mathematical skills. This approach allows readers to easily adapt these materials for their own educational efforts, even with limited prior experience.

3.1 Linear Algebra

1. Complex Numbers
   Let $\mathbb{R}$ and $\mathbb{C}$ denote the set of real numbers and the set of complex numbers, respectively. A *complex number* $c \in \mathbb{C}$ is written in its standard form as
   $$c = a + bi,$$
   where $a, b \in \mathbb{R}$, and i is the imaginary unit satisfying $i^2 = -1$.
   The *conjugate* of c is denoted by c^* and is given by
   $$c^* = a - bi,$$
   The *magnitude* or length or modulus of $c \in \mathbb{C}$ is
   $$|c| = \sqrt{c \cdot c^*} = \sqrt{a^2 + b^2}$$

2. Complex Vectors

Complex vectors shall be crucial since they represent quantum states (more details are discussed in section 3.2). Let $|\psi\rangle \in \mathbb{C}^d$ denote a complex (column) *vector*:

$$|\psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_d \end{pmatrix},$$

where each entry $\psi_i \in \mathbb{C}$ for $i = 1, 2, \ldots, d$.

The notation $|\cdot\rangle$ is referred to as *Dirac* notation, and is read as "ket". The dual representation of $|\psi\rangle$, denoted as $\langle\psi|$ and read as "bra," is the conjugate transpose (Hermitian conjugate) of $|\psi\rangle$. It is represented as a row vector:

$$\langle\psi| = (\psi_1^*, \psi_2^*, \ldots, \psi_d^*).$$

Exercise 3.1. Given $|\psi\rangle = \begin{pmatrix} 1 \\ i \end{pmatrix}$, what is $\langle\psi|$?

Solution: The conjugate of 1 is $1^* = 1$, and the conjugate of i is $i^* = -i$. Therefore, $\langle\psi| = (1 \quad -i)$.

3. Matrix Operations

Since we require some essential matrix operations, let us review them briefly. Let A be a matrix, and $A(i, j)$ represent the entry of A in the i-th row and j-th column. The following operations are defined as:

- **Conjugate of A:** $A^*(i, j) = (A(i, j))^*$.
- **Transpose of A:** $A^T(i, j) = A(j, i)$.
- **Adjoint (also called conjugate transpose, Hermitian conjugate, or dagger) of A:** $A^\dagger = (A^*)^T$.

Example 3.1. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Then, the transpose of A is given by:

$$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}.$$

Let A and B be two matrices. The dot product (multiplication) of A and B is given by

$$A \cdot B(i, j) = \sum_{k=1}^{d} A(i, k) \cdot B(k, j).$$

Example 3.2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix},$$

then

$$A \cdot B = \begin{pmatrix} 1 \cdot 4 + 2 \cdot 2 & 1 \cdot 3 + 2 \cdot 1 \\ 3 \cdot 4 + 4 \cdot 2 & 3 \cdot 3 + 4 \cdot 1 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 20 & 13 \end{pmatrix}.$$

Exercise 3.2. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad B = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}.$$

What is $A \cdot B$?

4. Inner Product

Given two vectors $|\psi\rangle$ and $|\phi\rangle$, the *inner product* of them is denoted by $\langle\psi|\phi\rangle$, and is defined as

$$\langle\psi|\phi\rangle = \sum_{i=1}^{d} \psi_i^* \cdot \phi_i.$$

The inner product measures the "overlap" (or similarity) between the two vectors. If $\langle\psi|\phi\rangle = 0$, then the vectors are orthogonal, and if $\langle\psi|\phi\rangle = 1$, then the vectors are aligned in the same direction.

Exercise 3.3. Let $|\psi\rangle = \begin{pmatrix} 1 \\ i \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} 2 \\ 3i \end{pmatrix}$. Compute their inner product.

Solution:

$$\langle\psi|\phi\rangle = (1 - i) \cdot \begin{pmatrix} 2 \\ 3i \end{pmatrix} = 1 \cdot 2 + (-i) \cdot (3i) = 2 + 3 = 5.$$

Note that the inner product returns a scalar, and we have

$$(\langle\psi|\phi\rangle)^* = \langle\phi|\psi\rangle.$$

5. Euclidean Norm

The *Euclidean norm (2-norm)* of a vector $|\psi\rangle$ is denoted by $\||\psi\rangle\|_2$, and is given by

$$\||\psi\rangle\|_2 = \sqrt{\langle\psi|\psi\rangle} = \sqrt{\sum_{i=1}^{d} \psi_i^* \psi_i} = \sqrt{\sum_{i=1}^{d} |\psi_i|^2}.$$

Exercise 3.4. Let $|\psi\rangle = \begin{pmatrix} 3 + 4i \\ 1 - i \end{pmatrix}$. Compute its Euclidean norm.

Solution:

$$\||\psi\rangle\|_2 = \sqrt{(3 + 4i)^*(3 + 4i) + (1 - i)^*(1 - i)} = 3\sqrt{3}.$$

6. Outer Product

For two vectors $|\psi\rangle, |\phi\rangle \in \mathbb{C}^d$, the *outer product* $|\psi\rangle\langle\phi|$ yields a $d \times d$ matrix.

Example 3.3. Let $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$|0\rangle\langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \quad 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$|1\rangle\langle 0| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (1 \quad 0) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Exercise 3.5. Exercise: Let $|\psi\rangle = \begin{pmatrix} 3 \\ 2 - i \end{pmatrix}$. What is $|\psi\rangle\langle\psi|$? **Solution**:

$$|\psi\rangle\langle\psi| = \begin{pmatrix} 3 \\ 2 - i \end{pmatrix} (3 \quad 2 + i)$$

$$= \begin{pmatrix} 9 & 6 + 3i \\ 6 - 3i & 4 - (i)^2 \end{pmatrix} = \begin{pmatrix} 9 & 6 + 3i \\ 6 - 3i & 5 \end{pmatrix}$$

7. Linear Operators

A *linear operator* A is a $d \times d$ matrix that maps $\mathbb{C}^d \to \mathbb{C}^d$ with the following linear property:

$$A\left(\sum_i a_i|\psi_i\rangle\right) = \sum_i a_i A|\psi_i\rangle,$$

where $a_i \in \mathbb{C}$ and $|\psi_i\rangle \in \mathbb{C}^d$.

The *matrix element* is a scalar quantity that provides information about how the operator A acts on the state $|\psi\rangle$ and how the resulting state overlaps with the state $|\phi\rangle$.

$$\langle\phi|A|\psi\rangle = \langle\phi|(A|\psi\rangle) = \sum_{i,j} \phi_i^*(A(i,j)\psi_j),$$

where ϕ_i^* is the complex conjugate of the i-th component of $|\phi\rangle$, $A(i,j)$ is the (i,j)-th entry of the operator A, and ψ_j is the j-th component of $|\psi\rangle$.

Example 3.4. Given $|\psi\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |\phi\rangle = \begin{pmatrix}0\\1\end{pmatrix}, A = \begin{pmatrix}2 & 3\\1 & 4\end{pmatrix}$,

$$\langle\phi|A|\psi\rangle = \langle\phi|\begin{pmatrix}2 & 3\\1 & 4\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} = 1.$$

Thus, $\langle\phi|A|\psi\rangle = 1$.

8. Orthonormal Bases

A set of vectors $\{|\psi_i\rangle\} \subseteq \mathbb{C}^d$ is said to be *orthogonal* if for all $i \neq j$, $\langle\psi_i|\psi_j\rangle = 0$. The set is *orthonormal* if

$$\langle\psi_i|\psi_j\rangle = \begin{cases}0, & \text{if } i \neq j,\\ 1, & \text{if } i = j.\end{cases}$$

Hence, each vector satisfies $\||\psi_i\rangle\|_2 = 1$, and every distinct pair of vectors is orthogonal.

For every vector in $\mathbb{C}^d$, it can be expressed as a linear combination of an orthonormal basis. For example, in $\mathbb{C}^2$, the most common basis is $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$. Any vector $|\psi\rangle \in \mathbb{C}^2$ can be written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where $\alpha, \beta \in \mathbb{C}$. We say that $|\psi\rangle$ is *normalized* if $\||\psi\rangle\|_2 = 1$, which is equivalent to

$$|\alpha|^2 + |\beta|^2 = 1.$$

Exercise 3.6. Why $\||\psi\rangle\|_2 = 1$ is equivalent to $|\alpha|^2 + |\beta|^2 = 1$?

Solution: $\||\psi\rangle\|_2 = \sqrt{\langle\psi|\psi\rangle} = \langle\alpha|0\rangle + \beta|1\rangle, \alpha|0\rangle + \beta|1\rangle = (\alpha^*\langle 0| + \beta^*\langle 1|)(\alpha|0\rangle + \beta|1\rangle) = |\alpha|^2\langle 0|0\rangle + \alpha^*\beta\langle 0|1\rangle + \beta^*\alpha\langle 1|0\rangle + |\beta|^2\langle 1|1\rangle = |\alpha|^2 + |\beta|^2 = 1$

9. Eigenvalues and Eigenvectors

Given a matrix A, an eigenvector $|\psi\rangle$ is a non-zero vector that satisfies the equation

$$A \cdot |\psi\rangle = \lambda|\psi\rangle$$

for some scalar $\lambda \in \mathbb{C}$. We call λ the corresponding eigenvalue of A.

Example 3.5. Show that $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is an eigenvector of $A = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$.

$$A|+\rangle = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}(|1\rangle + |0\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = |+\rangle$$

Thus, $|+\rangle$ is an eigenvector of A with eigenvalue $\lambda = 1$.

Exercise 3.7. Show that $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ is also an eigenvector of $A = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ and find the corresponding eigenvalue.

Solution:

$$A|-\rangle = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix} = -\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = -|-\rangle$$

Thus, $|-\rangle$ is an eigenvector of A with eigenvalue $\lambda = -1$.

We can find the eigenvalues of a matrix A without knowing the eigenvectors. The eigenvalues satisfy the characteristic equation:

$$\det(A - \lambda I) = 0$$

where det denotes the determinant and I is the identity matrix. Let us first review how to compute the determinant with some examples.

Example 3.6.

$$\begin{vmatrix}1 & 2\\2 & 4\end{vmatrix} = 1 \cdot 4 - 2 \cdot 2 = 0$$

$$\begin{vmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{vmatrix} = 1\cdot5\cdot9+2\cdot6\cdot7+3\cdot4\cdot8-3\cdot5\cdot7-1\cdot6\cdot8-2\cdot4\cdot9$$
$$= 45 + 84 + 96 - 105 - 48 - 72 = 0$$

Example 3.7. Let $A = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$. Compute its eigenvalues.

Solution:

$$\det(A - \lambda I) = 0$$
$$A - \lambda I = \begin{pmatrix}-\lambda & -i\\i & -\lambda\end{pmatrix}$$
$$\det(A - \lambda I) = (-\lambda)(-\lambda) - (-i)(i) = \lambda^2 - 1 = 0$$

$$\lambda^2 = 1$$
$$\lambda = \pm 1$$

3.2 Quantum Mechanics

With linear algebra at hand, the four postulates of quantum mechanics can be stated, addressing the following questions: How to represent a single quantum system, how to perform operations on a quantum system, how to describe multiple quantum systems, and how to measure classical information from a quantum system.

Postulate 1: Individual Quantum Systems

Recall that in classical computing, a bit is either 0 or 1. In the quantum world, a quantum bit, or *qubit*, can take on not just 0 or 1, but a state that reflects the possibility of being both 0 and 1 simultaneously. Let us formalize this phenomenon.

First, we encode the bits 0 and 1 via the standard orthonormal basis vectors $|0\rangle$ and $|1\rangle$ in $\mathbb{C}^2$. Then, to denote a qubit in states $|0\rangle$ and $|1\rangle$ simultaneously, we write:

$$|0\rangle + |1\rangle.$$

This is called a *superposition*.

More generally, the contribution of $|0\rangle$ and $|1\rangle$ is controlled by the *amplitudes* $\alpha, \beta \in \mathbb{C}$, i.e.,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

The only restriction is that $|\psi\rangle$ must be normalized (a unit vector), i.e.,

$$|\alpha|^2 + |\beta|^2 = 1.$$

In summary, any unit vector in $\mathbb{C}^2$ describes the state of a single qubit.

Example 3.8. $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ are two widely mentioned single-qubit states.

Postulate 2: Quantum Operations

Recall that a linear operator acts on the state vector and is represented by a matrix. The matrix must be with dimensions $d \times d$ to operate on a state in $\mathbb{C}^d$. A quantum operation is represented by a linear operator, which must be a *unitary* matrix. A matrix U is *unitary* if it satisfies the condition

$$UU^\dagger = U^\dagger U = I,$$

where $U^\dagger$ is the Hermitian conjugate (or adjoint) of U, and I is the identity matrix. Therefore, $U^\dagger$ is the inverse of U.

Example 3.9. *Pauli-X gate:*

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$X^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad XX^\dagger \quad = X^\dagger X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Pauli-Y gate:

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$Y^\dagger = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad YY^\dagger \quad = Y^\dagger Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Pauli-Z gate:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$Z^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad ZZ^\dagger \quad = Z^\dagger Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
$$H^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad HH^\dagger \quad = H^\dagger H = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Exercise 3.8.

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle,$$
$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle.$$

Hence, X is also called the quantum OR gate.

$$Y|0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i|1\rangle,$$
$$Y|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i|0\rangle.$$

$$Z|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle,$$
$$Z|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle.$$

$$Z|+\rangle = Z\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) = \frac{1}{\sqrt{2}}(Z|0\rangle + Z|1\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle.$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle,$$
$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle.$$

$$H|+\rangle = H\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) = \frac{1}{\sqrt{2}}(H|0\rangle + H|1\rangle)$$
$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) = |0\rangle.$$

$$H|-\rangle = H\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) = \frac{1}{\sqrt{2}}(H|0\rangle - H|1\rangle)$$
$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) = |1\rangle.$$

Postulate 3: Composite Quantum Systems

So far, we have only considered single-qubit systems. However, a computer with just a single qubit is not very useful. To perform more complex computations, we need to combine multiple qubits. The tool for this task is the *tensor product*, denoted by $\otimes$. Formally, for two vectors $|\psi\rangle$ and $|\phi\rangle$ in $\mathbb{C}^2$, the tensor product $|\psi\rangle \otimes |\phi\rangle$ is a vector in $\mathbb{C}^4$, with

$$(\psi \otimes \phi)_{ij} = \psi_i \phi_j.$$

This expresses that the (i, j)-entry of the tensor product $|\psi\rangle \otimes |\phi\rangle$ is the product of the i-th component of $|\psi\rangle$ and the j-th component of $|\phi\rangle$.

Example 3.10.

$$|0\rangle \otimes |0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$|0\rangle \otimes |1\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$|1\rangle \otimes |0\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$|1\rangle \otimes |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Note, that a 2-qubit system can exist in a superposition of 4 classical basis states, and an n-qubit system can exist in a superposition of 2^n classical basis states. Although it does not hold 2^n bits of information, its ability to exist in a superposition of states, combined with *entanglement*, is why a quantum computer might **potentially** outperform classical computers.

Now, let us look at a 2-qubit state that troubled Einstein until the end of his days, one of the *Bell* states:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Note that $|00\rangle$ is $|0\rangle \otimes |0\rangle$. This state demonstrates a quantum phenomenon known as *entanglement*. Intuitively, it means that if a pair of qubits q_0 and q_1 are entangled, then they are bound regardless of the distance between them, and one cannot describe the state of q_0 or q_1 alone. This means that there do not exist two states $|\psi_1\rangle$ and $|\psi_2\rangle$ in $\mathbb{C}^2$ such that

$$|\Phi^+\rangle = |\psi_1\rangle \otimes |\psi_2\rangle.$$

For $|\Phi^+\rangle$, when we measure it, the two qubits are either both $|00\rangle$ or both $|11\rangle$ since they are entangled (quantum measurement is discussed later in detail).

The other three Bell states are:

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle),$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

For a linear operator to qualify as a quantum gate, it must be a unitary operator. Moreover, for an n-qubit quantum system, the quantum gates are $2^n \times 2^n$ matrices. For example, we have seen that 2-qubit quantum states are described by unit vectors in $\mathbb{C}^4$. Accordingly, we can discuss 2-qubit quantum gates, which are unitary operators (matrices) of dimension 4×4. There are two types of such gates: tensor products of single-qubit gates and genuinely 2-qubit gates.

For a $d_1 \times d_1$ matrix A and a $d_2 \times d_2$ matrix B, the tensor product $A \otimes B$ results in a $d_1 d_2 \times d_1 d_2$ matrix.

Example 3.11. Let $A = \begin{pmatrix}a_1 & a_2\\a_3 & a_4\end{pmatrix}$ and $B = \begin{pmatrix}b_1 & b_2\\b_3 & b_4\end{pmatrix}$.

The tensor product $A \otimes B$ is given by:

$$A \otimes B = \begin{pmatrix}a_1 B & a_2 B\\a_3 B & a_4 B\end{pmatrix} = \begin{pmatrix}a_1b_1 & a_1b_2 & a_2b_1 & a_2b_2\\a_1b_3 & a_1b_4 & a_2b_3 & a_2b_4\\a_3b_1 & a_3b_2 & a_4b_1 & a_4b_2\\a_3b_3 & a_3b_4 & a_4b_3 & a_4b_4\end{pmatrix}.$$

Example 3.12.

$$X \otimes Z = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

$$H \otimes H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
$$= \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

Exercise 3.9. Compute $X \otimes I$, $Z \otimes H$.

Solution:

$$X \otimes I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$Z \otimes H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Genuinely 2-qubit gates are not tensor products of single-qubit gates. An important example of such a gate is the controlled-NOT (CNOT) gate. The CNOT gate treats the first qubit as the control qubit and the second as the target qubit. It applies the Pauli X gate (NOT gate) to the target qubit only if the control qubit is $|1\rangle$; otherwise, it does nothing. More precisely:

The CNOT gate is represented by the matrix:

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix},$$

The action of the CNOT gate on computational basis states is as follows:

$$\text{CNOT}\,|00\rangle = |00\rangle, \quad \text{CNOT}\,|01\rangle = |01\rangle,$$
$$\text{CNOT}\,|10\rangle = |11\rangle, \quad \text{CNOT}\,|11\rangle = |10\rangle.$$

Now, we can do our first interesting computation: we can prepare the Bell state $|\Phi^+\rangle$ starting from an initial state of two qubits $|0\rangle|0\rangle$ (or $|0\rangle \otimes |0\rangle$, denoted as $|00\rangle$).

Example 3.13.

$$\begin{aligned} \text{CNOT}\,(H \otimes I)|00\rangle &= \text{CNOT}\,(H|0\rangle \otimes I|0\rangle) \\ &= \text{CNOT}\,(|+\rangle \otimes |0\rangle) \\ &= \text{CNOT}\,\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle\right) \\ &= \text{CNOT}\,\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \\ &= \frac{1}{\sqrt{2}}(\text{CNOT}\,|00\rangle + \text{CNOT}\,|10\rangle) \\ &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \end{aligned}$$

Thus, the resulting state is the Bell state:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

Exercise 3.10. Try using the initial states $|01\rangle$, $|10\rangle$, and $|11\rangle$ with the same gates as in Exercise 3.13 to construct the other Bell states.

Postulate 4: Measurement

The measurement of a quantum state involves three classes of linear operators: Hermitian operators, positive semi-definite operators, and orthogonal projection operators. Readers can decide whether to cover these topics in their courses depending on the depth and audience of the course.

Without mentioning these operators, the measurement can be simplified as follows:

For a single-qubit state $\alpha|0\rangle+\beta|1\rangle$, the probability of measuring the outcome $|0\rangle$ is $|\alpha|^2$, and the probability of measuring the outcome $|1\rangle$ is $|\beta|^2$. After the measurement, the state collapses to the measured basis state, either $|0\rangle$ or $|1\rangle$.

For a two-qubit system in the state

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle,$$

the probabilities of measuring the outcomes $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ are $|\alpha|^2$, $|\beta|^2$, $|\gamma|^2$, and $|\delta|^2$, respectively. Upon measurement, the state collapses to the measured basis state corresponding to the outcome.

In general, for an n-qubit quantum system, the measurement outcomes are determined by the probabilities associated with the amplitudes of each computational basis state. These probabilities always sum to 1, ensuring the state is properly normalized.

4 Challenges and Solutions

With Sections 3.1 and 3.2, we have reduced the mathematics and physics barriers for students to understand quantum computing. Readers can proceed with the rest of their courses, exploring desired quantum "tricks" or algorithms. In this section, we summarize some of the challenges students may face when first learning quantum computing and offer potential solutions.

Terminologies, Notations, and Meanings

The first challenge students might encounter is that quantum computing, being a multidisciplinary topic, often uses multiple names for the same concept. For example, for a complex number c, $|c|$ is referred to as the length, magnitude, or modulus. Additionally, many distinct concepts may have similar names, such as dot product, inner product, outer product, and tensor product.

In fact, we have observed that some materials on quantum computing—and occasionally ChatGPT—claim that the outer product and tensor product for vectors are the same, which is incorrect. This can be very confusing for students.

To address this challenge, in Section 3, we provide distinct names for the same content during its definition, ensuring clarity. Furthermore, we include examples and exercises to help students become more familiar with these concepts and their differences.

Circuit Diagrams

Understanding the circuit diagrams of quantum operations can be confusing for students, even though they are similar to classical circuit diagrams. Here, we highlight some potentially confusing aspects for students and provide a famous example to clarify these concepts: quantum teleportation. In a quantum circuit diagram:

1. A single wire represents a single qubit state.
2. A single wire with no gate can be interpreted as applying the identity matrix I to the single qubit state.
3. Multiple single wires represent the tensor product of individual single-qubit states.
4. A double wire following a measurement indicates that the output of the measurement is a classical bit string.

What is quantum teleportation? Suppose there is a single-qubit system given by:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where the values of α and β are unknown. How can this state be transmitted to a friend?

Quantum teleportation utilizes an entangled Bell state, specifically:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

This state consists of two entangled qubits. Suppose the first qubit is held by you, and the second by your friend. Using the following quantum circuit, the state $|\psi\rangle$ can be 'teleported' from you to your friend.

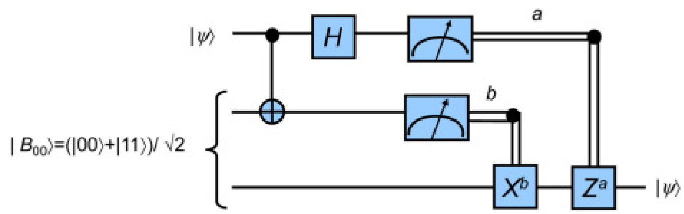

Figure 1: Quantum teleportation circuit diagram.

This means we initially start with the state:

$$|\psi\rangle \otimes |\Phi^+\rangle$$

where $|\psi\rangle$ is the state we want to teleport, and $|\Phi^+\rangle$ is a Bell state. Explicitly, this can be written as:

$$|\psi\rangle \otimes |\Phi^+\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}}\Big(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\Big)$$

We first apply the CNOT gate on the first qubits, and get

$$\frac{1}{\sqrt{2}}\Big(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\Big)$$

Then, the H gate is applied on the first qubit. So after applying the H gate, we have

$$\frac{1}{\sqrt{2}}\Big(\alpha|+\rangle \otimes |00\rangle + \alpha|+\rangle \otimes |11\rangle + \beta|-\rangle \otimes |10\rangle + \beta|-\rangle \otimes |01\rangle\Big)$$

$$= \frac{1}{2}\Big(\alpha(|0\rangle + |1\rangle) \otimes |00\rangle + \alpha(|0\rangle + |1\rangle) \otimes |11\rangle + \beta(|0\rangle - |1\rangle) \otimes |10\rangle + \beta(|0\rangle - |1\rangle) \otimes |01\rangle\Big)$$

$$= \frac{1}{2}\Big(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\Big)$$

This shows that the resulting 3-qubit state has 4 components, and when measured, each component has a probability of

$(\frac{1}{2})^2 = 25\%$ of being observed. The remarkable part is that the measurement of the first two qubits (held by you) determines the state of the third qubit (held by your friend).

If the first two qubits are measured as $|00\rangle$, then the third qubit held by your friend must be $|\psi\rangle$, completing the teleportation.

For other possible measurements:

1. If the first two qubits are measured as $|01\rangle$, your friend can apply an X gate to their qubit:
$$X(\alpha|1\rangle + \beta|0\rangle) = |\psi\rangle$$
2. If the first two qubits are measured as $|10\rangle$, your friend can apply a Z gate:
$$Z(\alpha|0\rangle - \beta|1\rangle) = |\psi\rangle$$
3. If the first two qubits are measured as $|11\rangle$, your friend can apply ZX gates:
$$ZX(\alpha|1\rangle - \beta|0\rangle) = |\psi\rangle$$

Thus, based on your measurement, your friend can always reconstruct the quantum state $|\psi\rangle$ instantaneously, regardless of the distance between you and your friend. However, this does not imply that information can be transmitted faster than the speed of light, as the result of your measurement must still be communicated to your friend through a classical channel.

Algebraic Rules

The final challenge addressed in this paper is the unfamiliarity with many algebraic rules used in quantum computing for students. As a result, even simple computations can cause hesitation.

To address this, we summarize some fundamental algebraic rules in quantum computing and demonstrate their simplicity and utility through a proof of the famous no-cloning theorem.

Let A, B, C, D be matrices and $|a\rangle$, $|b\rangle$, $|c\rangle$, $|d\rangle$ be vectors.

$$(AB)^\dagger = B^\dagger A^\dagger \tag{1}$$
$$(AB)^T = B^T A^T \tag{2}$$
$$\langle(\alpha|0\rangle + \beta|1\rangle)| = \alpha^*\langle 0| + \beta^*\langle 1| \tag{3}$$
$$(|a\rangle + |b\rangle) \otimes |c\rangle = |a\rangle \otimes |c\rangle + |b\rangle \otimes |c\rangle \tag{4}$$
$$|a\rangle \otimes (|b\rangle + |c\rangle) = |a\rangle \otimes |b\rangle + |a\rangle \otimes |c\rangle \tag{5}$$
$$\alpha(|a\rangle \otimes |b\rangle) = (\alpha|a\rangle) \otimes |b\rangle = |a\rangle \otimes (\alpha|b\rangle) \tag{6}$$
$$(|a\rangle \otimes |b\rangle)^\dagger = |a\rangle^\dagger \otimes |b\rangle^\dagger = \langle a| \otimes \langle b| \tag{7}$$
$$(\langle a| \otimes \langle c|)(|b\rangle \otimes |d\rangle) = \langle a|b\rangle\langle c|d\rangle \tag{8}$$
$$(A \otimes B)(C \otimes D) = (AC) \otimes (BD) \tag{9}$$
$$\mathrm{Tr}(A \otimes B) = \mathrm{Tr}(A)\,\mathrm{Tr}(B) \tag{10}$$

where Tr represents the trace of a matrix.

Example 4.1.

$$\begin{aligned}|1\rangle \otimes |-\rangle &= |1\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \\ &= \frac{1}{\sqrt{2}}|1\rangle \otimes |0\rangle - \frac{1}{\sqrt{2}}|1\rangle \otimes |1\rangle \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\-1\end{pmatrix}.\end{aligned}$$

With these rules at hand, let us prove the no-cloning theorem for quantum states.

Theorem 4.1. Given an arbitrary quantum state $|\psi\rangle \in \mathbb{C}^2$, there is no quantum circuit capable of creating an exact copy of $|\psi\rangle$.

Proof: Suppose there exists a unitary operator U of dimension 2×2 such that for any quantum state $|\psi\rangle \in \mathbb{C}^2$, U maps $|\psi\rangle \otimes |0\rangle$ to $|\psi\rangle \otimes |\psi\rangle$, i.e., U creates a copy of $|\psi\rangle$.

Then, for two arbitrary states $|\psi_1\rangle$, $|\psi_2\rangle$ in $\mathbb{C}^2$, we have:

$$|\phi\rangle = U(|\psi_1\rangle \otimes |0\rangle) = |\psi_1\rangle \otimes |\psi_1\rangle$$
$$|\phi'\rangle = U(|\psi_2\rangle \otimes |0\rangle) = |\psi_2\rangle \otimes |\psi_2\rangle$$

Now, consider the inner product $\langle\phi|\phi'\rangle$:

$$\begin{aligned}\langle\phi|\phi'\rangle &= (|\phi\rangle)^\dagger|\phi'\rangle \\ &= (U(|\psi_1\rangle \otimes |0\rangle))^\dagger\, U(|\psi_2\rangle \otimes |0\rangle) \\ &= (|\psi_1\rangle \otimes |0\rangle)^\dagger U^\dagger U(|\psi_2\rangle \otimes |0\rangle) \\ &= (\langle\psi_1| \otimes \langle 0|)(|\psi_2\rangle \otimes |0\rangle) \\ &= \langle\psi_1|\psi_2\rangle\langle 0|0\rangle \\ &= \langle\psi_1|\psi_2\rangle\end{aligned}$$

Also,

$$\begin{aligned}\langle\phi|\phi'\rangle &= (|\psi_1\rangle \otimes |\psi_1\rangle)^\dagger(|\psi_2\rangle \otimes |\psi_2\rangle) \\ &= (\langle\psi_1| \otimes \langle\psi_1|)(|\psi_2\rangle \otimes |\psi_2\rangle) \\ &= \langle\psi_1|\psi_2\rangle\langle\psi_1|\psi_2\rangle \\ &= \langle\psi_1|\psi_2\rangle^2.\end{aligned}$$

This implies $\langle\psi_1|\psi_2\rangle = \langle\psi_1|\psi_2\rangle^2$. Hence, $\langle\psi_1|\psi_2\rangle$ is either 0 or 1.

Therefore, $|\psi_1\rangle$ and $|\psi_2\rangle$ are either orthogonal or in the same direction. This leads to a contradiction. □

5 Evaluation and Conclusion

In this paper, we introduced an approach to teaching introductory quantum computing from a computer science perspective, aiming to lower the mathematical and physical barriers for students. In Section 3, we summarized nine key concepts in linear algebra and reformulated the four postulates of quantum mechanics using linear algebra, making the material more accessible for computer science students. While the course has not yet been offered, and its evaluation and analysis remain as future work, the framework presented here provides a practical and adaptable starting point. We hope that educators can use this approach to initiate their own efforts in quantum computing education, fostering broader accessibility and interest in this emerging field.

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